



MATHEMATICS

2C/2D

Calculator-free

WACE Examination 2011

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

When examiners design an examination, they develop provisional marking keys that can be reviewed at a marking key ratification meeting and modified as necessary in the light of candidate responses.

Question 1

(10 marks)

- (a) How many significant figures are there in the number 0.02070? (1 mark)

Solution
4
Specific Behaviours
✓ determines number of significant figures

- (b) Write 0.0038 in scientific notation. (1 mark)

Solution
3.8×10^{-3}
Specific Behaviours
✓ expresses answer in scientific notation correctly

- (c) Factorise $4x^2 - 9$ (1 mark)

Solution
$(2x + 3)(2x - 3)$
Specific Behaviours
✓ factorises expression correctly

- (d) Determine the gradient of the straight line given by $7x + 2y = 4$. (1 mark)

Solution
$-\frac{7}{2}$
Specific Behaviours
✓ determines gradient correctly

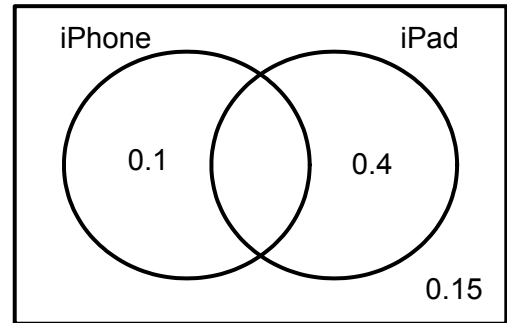
- (e) Given $f(x) = 5 - x^2$, determine the value of $f(-2)$. (1 mark)

Solution
1
Specific Behaviours
✓ evaluates function when $x = -2$

- (f) Write a recursive definition for the following sequence:
1, 1, 2, 3, 5, ... (1 mark)

Solution
$t_n = t_{n-1} + t_{n-2}$ With $t_0 = t_1 = 1$
Specific Behaviours
✓ states correct recursive definition with boundary conditions.

- (g) A group of students was surveyed about ownership of iPads and iPhones. The Venn diagram shows some of the probabilities associated with the responses.



Determine $P(\overline{iPad})$.

(1 mark)

Solution
$P(\overline{iPad}) = 0.25$
Specific Behaviours
✓ determines probability correctly

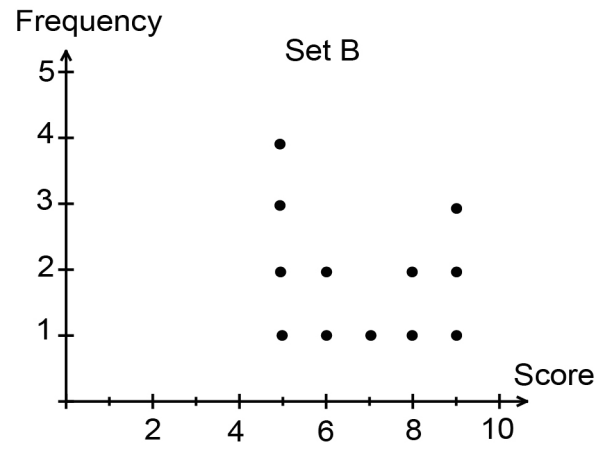
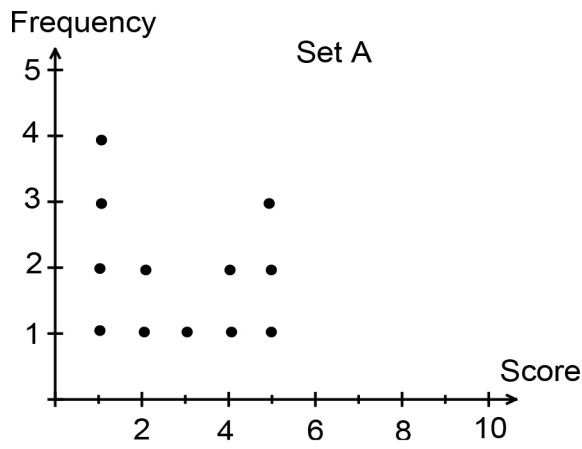
- (h) For the cubic function $y = (x + 2)(x - 3)(x + 1)$, determine the coordinates of the y -axis intercept (sketch not required). (1 mark)

Solution
$(0, -6)$
Specific Behaviours
✓ determines coordinates for y -axis intercept correctly

- (i) The following is a set of scores in ascending order: 4, 4, 6, x , 9, 11. If the median is 7, determine the value of the x . (1 mark)

Solution
$x = 8$
Specific Behaviours
✓ calculates correct value of x

- (j) Is the standard deviation of set A larger, smaller or the same as the standard deviation of set B? (1 mark)



Solution
Same
Specific Behaviours
✓ states that the standard deviations are the same

Question 2

(7 marks)

- (a) Simplify the expression $7x - 2(3 - 4x) - 5x$.

(2 marks)

Solution
$7x - 6 + 8x - 5x = 10x - 6$.
Specific Behaviours
<ul style="list-style-type: none"> ✓ expands brackets correctly ✓ simplifies expression

- (b) Solve the equation $18 - x^2 = 7x$.

(3 marks)

Solution
$x^2 + 7x - 18 = 0$ $(x - 2)(x + 9) = 0$ $x = 2$ or $x = -9$
Specific Behaviours
<ul style="list-style-type: none"> ✓ rearranges equation in the form $f(x) = 0$ ✓ factorises quadratic correctly ✓ solves equation to find both values of x <p>Or</p> <ul style="list-style-type: none"> ✓✓ uses guess and check for each solution ✓ shows relevant working

- (c) Determine the value of n given $\frac{5^3 \times 5^n}{5^4} = 5^6$.

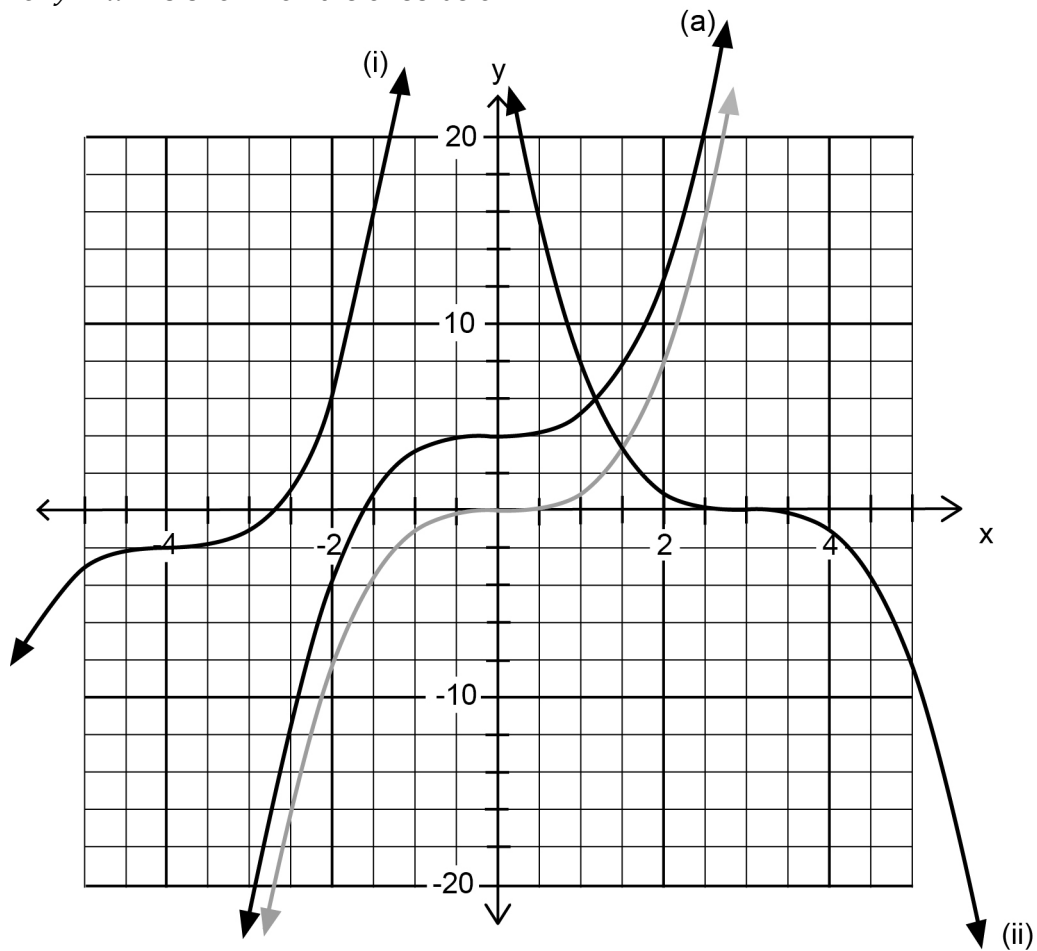
(2 marks)

Solution
$5^{n-1} = 5^6$ $n - 1 = 6$ or $\frac{5^n}{5} = 5^6$ $n = 7$ $5^n = 5^6 \therefore n = 7$
Specific Behaviours
<ul style="list-style-type: none"> ✓ simplifies the expression using index laws ✓ determines correct value of n

Question 3

(5 marks)

The graph of $y = x^3$ is shown on the axes below.



(a) Describe how the graph of $y = x^3$ can be used to draw the graph $y = x^3 + 4$.

(1 mark)

Solution
Vertical translation – move up 4 units (or 2 spaces) on scale (Graph shown but not needed) .
Specific Behaviours
✓ describes transformation as either a vertical translation or a shift upward by 4 units

(b) On the set of axes above sketch each of the following:

(i) $y = (x + 4)^3 - 2$ (2 marks)

Solution
On diagram above
Specific Behaviours
✓ sketches shape of graph correctly ✓ sketches graph accurately (location of point of inflection and reasonably congruent shape)

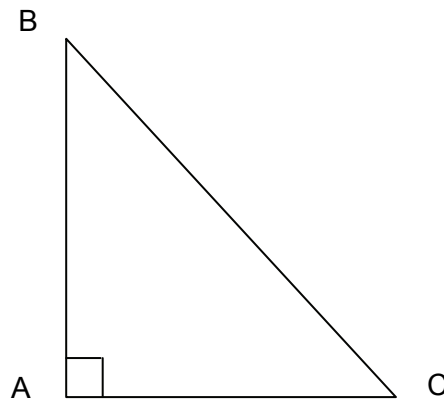
(ii) $y = -(x - 3)^3$ (2 marks)

Solution
On diagram Note: sketches should be labelled (i) and (ii).
Specific Behaviours
✓ sketches with correct orientation ✓ locates point of inflection correctly

Question 4

(5 marks)

Consider the triangle ABC where $\tan \angle ABC = \frac{3}{4}$. (Note the diagram is not drawn to scale.)



- (a) If the length of the side AB is 400 metres, calculate the length of the side AC. (3 marks)

Solution
$\tan \angle ABC = \frac{3}{4} = \frac{AC}{AB}$ $\therefore AC = \frac{3}{4} AB$ $= \frac{3}{4} (400)$ $= 300 \text{ m}$
Specific Behaviours
<ul style="list-style-type: none"> ✓ states the tangent ratio $\left(\tan \angle ABC = \frac{AC}{AB} = \frac{3}{4} \right)$ ✓ substitutes $AB = 400$ ✓ states correct value for AC

- (b) Determine the value of $\sin \angle BCA$. (2 marks)

Solution
$\sin \angle BCA = \frac{AB}{BC}$ $= \frac{400}{500}$ $= 0.8$ <p>(Accept $\frac{4}{5}$)</p>
Specific Behaviours
<ul style="list-style-type: none"> ✓ identifies correct sides ✓ determines correct value

Question 5

(4 marks)

Jacqui was testing her conjecture that:

‘for any three consecutive positive even numbers, the product of the first and third numbers is equal to the second number squared.’

- (a) Show that Jacqui’s conjecture does not work. (1 mark)

Solution
Answers may vary. For example, using 2, 4, and 6. $2 \times 6 = 12 \neq 4^2 = 16$
Specific Behaviours
✓ tests a set of three consecutive even numbers that disprove the conjecture

- (b) Test the conjecture for **two (2)** more sets of three consecutive positive even numbers. (2 marks)

Solution
Answers may vary. For example, using 4, 6 and 8 $4 \times 8 = 32 \neq 6^2 = 36$
Similarly, using 6, 8, and 10 $6 \times 10 = 60 \neq 8^2 = 64$
Specific Behaviours
✓✓ tests conjecture correctly for two other sets of three consecutive even positive numbers

- (c) Jacqui realised that she could revise her conjecture so that it is true. Her revised conjecture is:

‘for any three consecutive positive even numbers, the product of the first and third numbers is equal to _____ the middle number squared take 4. _____.’
Complete her revised conjecture. (1 mark)

Solution
The middle number squared take/minus/less four.
The square of the middle number take/minus/less four.
Specific Behaviours
✓ completes conjecture correctly

Question 6

(9 marks)

In a class of 32 students, of which 14 are females, it is found that three males are taking Biology. Fifteen students in the class are enrolled in Biology.

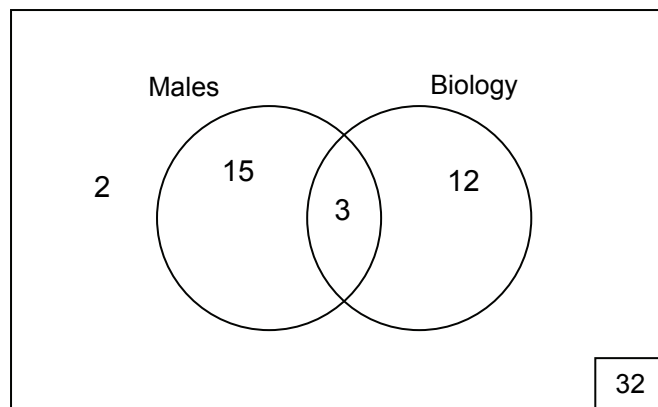
- (a) Express the above data in a two-way table and complete the missing entries.(3 marks)

Solution			
	Biology	Not in Biology	Row Totals
Males	3	15	18
Females	12	2	14
Column Totals	15	17	32

Specific Behaviours

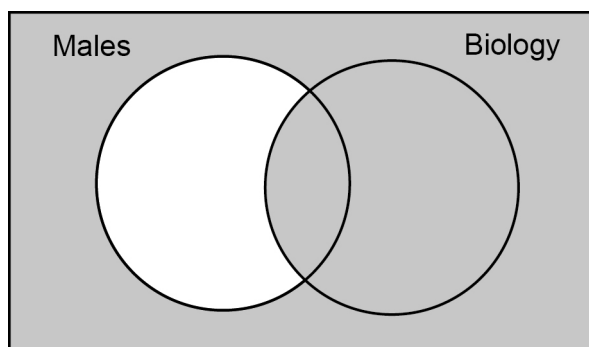
- ✓ constructs table with appropriate headings
- ✓ calculates entries for cells of the table
- ✓ sums entries correctly to determine totals

- (b) The above data can also be expressed as a Venn diagram. Complete the Venn diagram below by writing the relevant number of students in every region. (2 marks)



Solution
On diagram
Specific Behaviours
✓✓ determines correct numbers for each of the four regions
✓ determines numbers for two or three of the regions

- (c) Let M represent the set of Males and B represent the set of Biology students. Shade the region in the Venn diagram drawn below that is represented by the set $\bar{M} \cup B$. (1 mark)



Solution
shading on diagram above diagram
Specific Behaviours
✓ shades region correctly

- (d) Determine $P(\bar{M} \cup B)$. (1 mark)

Solution
$\frac{17}{32}$
Specific Behaviours
✓ determines probability correctly (based on response in part (b) and/or (c))

- (e) In the context of the question, describe the meaning of $n((M \cup B) | \bar{M})$. (2 marks)

Solution
$n((M \cup B) \bar{M})$ The number of students that are female and take biology.
Specific Behaviours
<ul style="list-style-type: none"> ✓ states that the 'n' represents the <u>number</u> of students ✓ states that the students are female and studying biology or other correct interpretation